

# Algebra II

## Grading

Homework: 35%

Tests: 65% (Tests may be retaken once and retake grades are capped at 89%)

0. What is math? (PowerPoint with discussion)

I. Fundamental Algebra Axioms

- a. Definition of all sorts of things that we've taken for granted (operations, closure, identity elements, inverse elements... we basically define a field, but never use the word field)
- b. Caley tables and wacky algebraic structures...brief introduction to the concept of "groups".
- c. Factoring and factor sets
  - i."Traditional" factoring of integers (what does it mean?)
  - ii. Factoring integers over wacky factor sets with explanation of the importance of convention in mathematics.

II. Linear Functions and Relations

- a. Functions and Relations
- b. Linear inequalities in one variable.
  - i. Graphs
  - ii. Compound inequalities.
- c. The Piece-Wise Function definition of absolute value, order and equivalent expressions
- d. The greatest Integer Function
- e. The Analytic (Euclidean) Geometry of Lines
  - i. equations of lines in various forms
  - ii. slopes of lines that are parallel, perpendicular
- f. Direct Variation with lots of modeling.

III. Systems of linear equations in multiple variables

- a. Modeling with multiple variables
- b. Geometrical Interpretations
  - i. Graphs of planes in "space"
  - ii. Types of solutions of systems of linear equations in three variables (points, lines and planes) with graphical and algebraic forms of solution and their relationships.
- c. Systems of linear inequalities in 2 variables.
  - i. algebraic method of solution
  - ii. Geometric interpretation of solutions (traces, projections of points, etc)
- d. Linear programming: Maximization problems subject to systems of linear constraints.

- i. The meaning of feasibility regions (geometric and conceptual interpretations)
- ii. Determining the maximum value of a linear function of two variables subject to the system of constraints with emphasis on geometrical interpretation (as section of plane within region)
- iii. Mathematical modeling with linear programming (“word problems”)

#### IV. Non-linear Algebra

- a. Review of laws of exponents with emphasis on derivations
- b. Review of multiplying polynomials and solving by factoring quadratics.
- c. Algebraic manipulations of Polynomials
  - i. factorization, expression as sums by long division
  - ii. exploration of why the long division algorithm (that everyone knows and no one understands) works.
    - i. Difference of Squares, Difference of Cubes, Sum of Cubes, *No Sum of Squares formula!*
    - ii. Ridiculous amounts of practice solving polynomial equations by factoring.
    - iii. Modeling!!! Emphasis on types of questions that appear frequently on standardized tests.
- d. Solving polynomial inequalities. emphasis on the conditions for  $ab > 0$  or  $ab < 0$  ( a simpler model which is easy to solve which guides analysis for products of binomials)
  - i. Higher order (factorable) polynomial inequalities (With number-line method of solution)
  - ii. Modeling with inequalities.

Interesting Aside: Group theory primer with mappings, onto, one-to-one and “operation”. Definition of group with clock arithmetic and composition of functions as examples)

#### V. Radicals and Complex Numbers

- a. Power function and “variation”
  - i. this and such “varies” as that and such or “is proportional to”...
  - ii. Even and odd functions
- b. Rational roots theorem
- c. Who put the “imaginary” in imaginary number? It’s really just slander that stuck. Axioms!
- d. Fractional exponents and equations involving radicals
- e. Addition, multiplication, division of Complex numbers and factorization over the set of polynomials with complex coefficients (So there really is a sum of squares formula!).

#### VI. Completing the Square (and the derivation of the famous quadratic formula)

- a. Completing the square with modeling
- b. The discriminant
- c. Roots and coefficients of a quadratic equation
- d. Graphing parabolas by completing the square (no more time-wasting memorization)

- e. Quadratic inequalities
- f. The fundamental theorem of algebra and conjugate pair roots of polynomials.

VII. Logarithms, Exponential Growth and Decay, Carbon Dating. Intro to Isomorphism

- a. Fractional exponent review, Function composition review and Inverse Functions
- b. Logarithms, bases and laws
- c. Solving exponential equations and change of base.
- d. Natural Logarithms
- e. Depending on time and interest: Review of Group-theory and intro to the concept of Isomorphism with logarithm example. The historic significance of this isomorphism (multiplying large numbers and slide-rules)

VIII. Matrices and Determinants

Depending on the interests of the class and the amount of time that we have remaining, we will cover some combination of the following topics:

IX. Analytic geometry (They have seen some of this in geometry)

- a. Distance formula
- b. Circles, Ellipses, Parabolas, Hyperbolas (focus-directrix definition)

X. Sequences and Series

XI. Permutations, Combinations and Probability